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On formal solutions of the Hormander's initial-boundary value problem in the class of Laurent series

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The derivation of the ring of Laurent series with supports in rational cones are defined and for polynomial differential operators with constant coefficients a theorem of existence and uniqueness of the solution of the analogue of one initial-boundary value problem of Hormander in the class of formal Laurent series is proved.

Keywords: differential operator, the Hormander's problem, difference equations, multiple Laurent series.

1. Introduction

A large number of papers have been devoted to various versions of the generalization of the Cauchy-Kovalevskaya theorem. Note the paper [1], which deals with the equation solved for mixed derivative, and also papers [2], [3], in which systems of linear partial differential equations and existence and uniqueness theorems for normal and analytic solutions of these systems were considered. In the paper of L. Hormander [4], in connection with the investigation of the Cauchy problem, the following initial-boundary value problem is formulated.

Let us consider a polynomial differential operator of the form

$$P(\partial,\xi)=\partial^m-\sum_{||\omega||\leqslant d}c_\omega(\xi)\partial^\omega,$$

where $\omega=(\omega_1,\ldots,\omega_n)$, $m=(m_1\ldots,m_n)$ are multi-indexes, $||\omega||=\omega_1+\cdots+\omega_n$, $||m||=d$, $\partial=(\partial_1,\ldots,\partial_n)$, ∂_j are partial derivative, the coefficients $c_\omega(\xi)$ are analytic functions of $\xi=(\xi_1,\ldots,\xi_n)$ in a neighborhood of zero in the space $\mathbb{C}^n$. It is required to find a function satisfying the differential equation

$$P(\partial,\xi)F=G, \tag{1}$$

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and conditions:

$$\partial_j^k [F - \Phi]|_{\xi_j=0} = 0, \;\; 0 \leqslant k < m_j, \;\; j = 1, \ldots, n, \tag{2}$$

where Φ and G are given analytic functions in a neighborhood of zero. In [4], Theorem 5.1.1, under certain restrictions on the coefficients $c_\omega(\xi)$ a theorem on the existence and uniqueness of an analytic solution of the initial-boundary value problem is obtained. The corollary to this theorem is both the Cauchy-Kovalevskaya theorem and the Darboux-Goursat-Baudot theorem.

In this paper we formulate an analogue of the problem (1)–(2), in which instead of the partial derivatives ∂_j the operators D_j derivation of the ring of Laurent series with supports in rational cones are used. In the case of constant coefficients, the question of the existence and uniqueness of a formal solution in the class of such series is investigated.

In the second section we give the notation and definitions necessary for the formulation of an analogue of the Hormander problem (1)–(2), and for the formulation of the theorem of existence and uniqueness of the solution of this problem (Theorem 1).

In the third section we define the Borel transform of Laurent series with supports in rational cones, which allows us to establish the connection (Theorem 2) between the solvability of the corresponding initial boundary value problems for differential and difference equations with identical symbols. Using the results on the solvability of the difference problem obtained in [5], we prove the main Theorem 1.

2. An analogue of the initial-boundary value problem of Hormander

To formulate an analogue of the initial-boundary value problem of Hormander, we need the definitions of a rational cone, of the ring of Laurent series with supports in these cones and derivations of the ring of such series (see [6],[7]).

Let $a^1, \ldots, a^n$ be *linearly independent* vectors with integer coordinates $a^j = (a_1^j, \ldots, a_n^j)$. *A rational cone* generated by $a^1, \ldots, a^n$, we call the set

$$K = \{x \in \mathbb{R}^n : x = \lambda_1 a^1 + \cdots + \lambda_n a^n, \lambda_j \in \mathbb{R}_+, j = 1, \ldots, n\}.$$

We denote by A the matrix whose columns consist of the coordinates of the vectors a^j and $\Delta = \det A$. If $\Delta = \det A$, then the cone K is unimodular. We shall consider only unimodular cones.

Let A^{-1} be the inverse of the matrix A. The rows of A^{-1} are denoted by $\alpha^1, \ldots, \alpha^n$. Let the vectors $\alpha^1, \ldots, \alpha^n$ form a *reciprocal basis* for the vectors $a^1, \ldots, a^n$, that is, $\langle \alpha^i, a^j \rangle = \delta_{ij}$, where $\langle k, x \rangle = k_1 x_1 + \cdots + k_n x_n$,, and δ_{ij} is the Kronecker symbol. We note that for $x \in K$ there is always $\langle \alpha^j, x \rangle \geqslant 0$, $j = 1, \ldots, n$.

Define the partial order $\underset{K}{\geqslant}$ between the points $u, v \in \mathbb{R}^n$ as follows:

$$u \underset{K}{\geqslant} v \Leftrightarrow u - v \in K.$$

Moreover, we write $u \underset{K}{\ngeqslant} v$ whenever $u - v \notin K$.

The cone

$$K^* = \{k \in \mathbb{R}^n : \langle k, x \rangle \geqslant 0, x \in K\}$$

is called dual to K. Denote the set of its interior points by $\overset{\circ}{K}{}^*$ and fix $\nu \in \overset{\circ}{K}{}^* \cap \mathbb{Z}^n$. Given $x \in K \cap \mathbb{Z}^n$, the nonnegative number

$$|x|_\nu = \langle \nu, x \rangle,$$

is referred to as the weighted homogeneous degree of z^x. The weighted homogeneous degree of the Laurent polynomial $Q(z) = \sum\limits_x q_x z^x$ is defined by the formula

$$\deg_\nu Q(z) = \max_x |x|_\nu.$$

Denote by $\mathbb{C}_K[z]$ the ring of Laurent polynomials $Q(z) = \sum\limits_x q_x z^x$ with the exponents x of the monomials z^x lying in $K \cap \mathbb{Z}^n$. Addition and multiplication are defined naturally.

In the ring $\mathbb{C}_K[[z]]$ of Laurent series the usual partial derivative $\frac{\partial}{\partial z_j}$ is not a derivation because for $x \in K \cap \mathbb{Z}^n$ the points $x - e^j$, where e^j are the standard unit vectors, in general need not lie in $K \cap \mathbb{Z}^n$. In [7], the derivation of the ring $\mathbb{C}_K[[z]]$ was determined, which made it possible to transfer the concept of D-finiteness of power series to Laurent series. We give this definition.

We can express each element $x \in K \cap \mathbb{Z}^n$ as a linear combination $x = \lambda_1 a^1 + \dots + \lambda_n a^n$ of basis vectors. In matrix form this expression becomes $x = A\lambda$, where λ is a column vector, and A is the matrix with the coordinates of the vectors a^j in columns

$$A = \begin{pmatrix} a_1^1 & \dots & a_1^n \\ .. & .. & .. \\ a_n^1 & \dots & a_n^n \end{pmatrix}.$$

Determinant of matrix A is not equal to zero.

Define the operator D_j on the monomials z^x as follows:

$$D_j z^x = \lambda_j z^{x - a^j}, \tag{3}$$

where λ_j is the j-th coordinate of the point $\lambda = A^{-1}x$. Observe that for $\Delta \neq 1$ and $x \in K \cap \mathbb{Z}^n$ the number λ_j is rational in general.

It is not difficult to verify that D_j for $j = 1, \dots, n$, are linear and satisfy the usual rule for the derivative of the product and, in the case of a unimodular cone ($\Delta = 1$), map the ring $\mathbb{C}_K[[z]]$ into itself, that is, they are derivations.

Next on the monomials z^x, $x \in K \cap \mathbb{Z}^n$, define the operator D^ω, $\omega \in K \cap \mathbb{Z}^n$, $\omega = \omega_1 a^1 + \dots + \omega_n a^n$ as follows:

$$D^\omega z^x = \Pi_j \langle x, \alpha^j \rangle \langle x - a^j, \alpha^j \rangle \dots \langle x - (\omega_j - 1) a^j, \alpha^j \rangle z^{x - \omega}.$$

Note that $D^{\omega'} D^{\omega''} = D^{\omega' + \omega''}$. For $\omega = a^j$ we have $D^{a^j} z^x = \langle x, \alpha^j \rangle z^{x - a^j} = D_j z^x$, $j = 1, \dots, n$, and if $\omega_1, \dots, \omega_n$ are the coordinates of the vector ω in the basis $a^1, \dots, a^n$, then $D^\omega = D^{a^1 \omega_1} \dots D^{a^n \omega_n} = D_1^{\omega_1} \dots D_n^{\omega_n}$, where $D_j^k = \underbrace{D_j \circ \dots \circ D_j}_{k\ times}$.

Note that if the cone K is unimodular, then the operators D^ω for $\omega \in K \cap \mathbb{Z}^n$ are derivations on the ring of series $\mathbb{C}_K[[z]]$ and the action of the operator D^ω on the monomials z^x, $x \in K \cap \mathbb{Z}^n$, can be written as follows:

$$D^\omega z^x = \begin{cases} 0, & \text{if } x \underset{K}{\not\geqslant} \omega, x \neq \omega, \\ \frac{\langle x, \alpha \rangle !}{\langle x - \omega, \alpha \rangle !} z^{x - \omega}, & \text{if } x \underset{K}{\geqslant} \omega. \end{cases} \tag{4}$$

where $\langle x, \alpha\rangle! = \langle x, \alpha^1\rangle! \ldots \langle x, \alpha^n\rangle!$.

We denote by Γ_j the face of the cone K generated by the vectors a^i, $i = 1, \ldots, j-1, j+1, \ldots, n$, $\Gamma_j = \{x : x = \lambda_1 a^1 + \ldots \lambda_{j-1} a^{j-1} + \lambda_{j+1} a^{j+1} + \cdots + \lambda_n a^n\}$. Denote $\mathcal{F}(z)|_{z^{a^j}=0}$ by the Laurent series, whose supports are the faces Γ_j of the rational cone K:

$$\mathcal{F}(z)|_{z^{a^j}=0} = \sum_{x \in \Gamma_j \cap \mathbb{Z}^n} f(x) z^x. \tag{5}$$

Let us define a polynomial differential operator

$$P(D) = \sum_{\omega \in \Omega} c_\omega(z) D^\omega, \tag{6}$$

where $\Omega \subset K \cap \mathbb{Z}^n$ is a finite set of points of an n-dimensional integer lattice and coefficients $c_\omega \in \mathbb{C}_K[[z]]$. The Laurent polynomial $P(\zeta) = \sum\limits_{\omega \in \Omega} c_\omega \zeta^\omega$ is called the characteristic polynomial for the polynomial differential operator (6).

By the order d_ν of $P(D)$, we mean the weighted homogeneous degree $\deg_\nu P(\zeta)$ of the characteristic polynomial, i.e., $d_\nu = \max\limits_{\omega \in \Omega} |\omega|_\nu$. In what follows we omit the subscript ν for d.

Remark. It follows from the relation

$$z^{a^j} D_j = \langle \alpha^j, \Theta\rangle, j = 1, \ldots, n, \tag{7}$$

where $\Theta = (z_1 \frac{\partial}{\partial z_1}, \ldots, z_n \frac{\partial}{\partial z_n})$, α^j, $j = 1, \ldots, n$, is a reciprocal basis, that the differential operator (6) can also be regarded as a differential operator with partial derivatives. As an example, consider the following operator:

$$P(D) = c_{2,1} D^{(2,1)} + c_{1,0} D^{(1,0)} + c_{1,1} D^{(1,1)} + c_{0,0}.$$

Considering (7), it can be written as

$$c_{2,1} z^{-1} \frac{\partial^2}{\partial z_1 \partial z_2} - c_{2,1} z_1^{-2} z_2 \frac{\partial^2}{\partial z_2^2} + c_{1,0} \frac{\partial}{\partial z_1} + (-c_{2,1} z_1^2 - c_{1,0} z_1^{-1} z_2 + c_{1,1} z_1^{-1}) \frac{\partial}{\partial z_2} + c_{0,0}.$$

Let us fix $m \in K \cap \mathbb{Z}^n$ and formulate the following problem which we call an analog of the initial-boundary value problem of Hormander. It is required to find $\mathcal{F}(z)$, satisfying the equation

$$P(D)\mathcal{F} = G, \tag{8}$$

and initial-boundary conditions

$$D^{a^j k}[\mathcal{F} - \Phi]|_{z^{a^j}=0} = 0, \;\; 0 \leqslant k < \langle m, \alpha^j\rangle, \;\; j = 1, \ldots, n. \tag{9}$$

For $K = \mathbb{R}^n_+$, $D_j = \partial_j$, $j = 1, \ldots, n$, the problem (8)–(9) was formulated in [4] and under certain restrictions on the coefficients of the homogeneous component of the highest power, the existence and uniqueness theorem is proved.

In the following theorem, in the case of constant coefficients of the polynomial differential operator $P(D)$, a sufficient condition is imposed on the coefficients of the principal symbol $P_d(D) = \sum\limits_{|\omega|_\nu = d} c_\omega D^\omega$ of the differential operator $P(D)$, which ensures the existence and uniqueness of the solution.

Theorem 2.1 *Let $m \in \Omega$ and let $|m|_\nu = d$ be the order of a differential operator. If the coefficients of operator (6) are constant and the coefficients of the principal symbol of $P_d(D)$ satisfy the condition*

$$|c_m| > \sum_{|\omega|_\nu = d, \omega \neq m} |c_\omega|, \tag{10}$$

then for any series $G, \Phi \in \mathbb{C}_K[[z]]$, the boundary value problem (8)–(9) has a unique solution $\mathcal{F} \in \mathbb{C}_K[[z]]$.

3. Proof of the main theorem

In this section we establish a connection between the solutions $\mathcal{F}$ of a differential equation from the ring $\mathbb{C}_K[[z]]$ and solutions of the difference equation.

In the one-dimensional case, the differential operator is written in the form $P(D) = \sum\limits_{\omega=0}^{m} c_\omega D^\omega$, $D = \frac{d}{dz}$, and the problem (8)–(9) takes the form

$$P(D)\mathcal{F}(z) = G(z) \tag{11}$$

$$D^\omega[\mathcal{F} - \Phi]|_{z=0} = 0, \quad 0 \leqslant \omega < m,$$

or, which is the same

$$\mathcal{F}^{(\omega)}(0) = \varphi(\omega), \omega = 0, 1, \ldots, m-1, \tag{12}$$

where $\varphi(\omega)$ are the coefficients of expansion in a series of functions $\Phi(z) = \sum\limits_{x=0}^{\infty} \frac{\varphi(x)}{x!} z^x$. If $G(z) = \sum\limits_{x=0}^{\infty} \frac{g(x)}{x!} z^x$, then it is not difficult to see that the function $\mathcal{F}(z) = \sum\limits_{x=0}^{\infty} \frac{f(x)}{x!} z^x$ is a solution of (11)–(12) if and only if the coefficients $f(x)$ in its expansion in a power series satisfy the difference equation

$$\sum_{\omega=0}^{m} c_\omega f(x+\omega) = g(x), x = 0, 1, \ldots, \tag{13}$$

with initial data

$$f(x) = \varphi(x), x = 0, 1, \ldots, m-1, \tag{14}$$

where $\varphi(x)$ is a given function.

Note that the generating function $F = \sum\limits_{x=0}^{\infty} \frac{f(x)}{z^x}$ of the solution of the difference equation (13) is the Borel transform of the solution $\mathcal{F}(z) = \sum\limits_{x=0}^{\infty} \frac{f(x)}{x!} z^x$ of the differential equation (11).

We formulate a multidimensional analogue of the difference problem. (13)–(14). On complex-valued functions $f(x) = f(x_1, \ldots, x_n)$ of integer variables $x_1, \ldots, x_n$, we define shift operators δ_j with respect to variables x_j:

$$\delta_j f(x) = f(x_1, \ldots, x_{j-1}, x_j + 1, x_{j+1}, \ldots, x_n)$$

and a polynomial difference operator of the form

$$P(\delta) = \sum_{\omega \in \Omega} c_\omega \delta^\omega,$$

where $\Omega \subset K \cap \mathbb{Z}^n$ is a finite set of points of an n-dimensional integer lattice, $\delta^\omega = \delta_1^{\eta_1} \cdot \dots \cdot \delta_n^{\eta_n}$, $\omega = \eta_1 e^1 + \dots + \eta_n e^n$ and coefficients $c_\omega \in \mathbb{C}_K[[z]]$.

It is required to find a function $f(x)$ satisfying a polynomial difference equation

$$P(\delta) f(x) = g(x) \tag{15}$$

and initial-boundary conditions

$$\delta^{a^j k}[f(x) - \varphi(x)]|_{x \in \Gamma_j \cap \mathbb{Z}^n} = 0, \ \ 0 \leqslant k < \langle m, \alpha^j \rangle, \ \ j = 1, \dots, n, \tag{16}$$

where Γ_j is the face of the cone K generated by the vectors a^i, $i = 1, \dots, j-1, j+1, \dots, n$, the vectors $\alpha^1, \dots, \alpha^n$ form a reciprocal basis for the vectors $a^1, \dots, a^n$, that is, $\langle \alpha^i, a^j \rangle = \delta_{ij}$.

Let us state the assertion (see [5], Theorem 1]), which is necessary for the proof of the main result.

Let c_ω are constant, $m \in \Omega$ and let $|m|_\nu = d$ be the order of a difference operator. If the coefficients c_ω of the principal symbol $P_d(\delta)$ satisfy the condition

$$|c_m| > \sum_{|\omega|_\nu = d, \omega \neq m} |c_\omega|,$$

then (15)–(16) is solvable.

For the case $K \cap \mathbb{Z}^n = \mathbb{Z}^n_+$, Theorem 2 was proved in [8].

For a function $f(x)$ of a discrete argument, we define two types of generating series (functions):

$$\mathcal{F}(z) = \sum_{x \in K \cap \mathbb{Z}^n} \frac{f(x) z^x}{\langle x, \alpha \rangle !}. \tag{17}$$

and

$$F(z) = \sum_{x \in K \cap \mathbb{Z}^n} \frac{f(x)}{z^x}. \tag{18}$$

A series (18) is called the Borel transform of a series (17). For $K = \mathbb{R}^n_+$, the standard Borel transform is obtained from the transformation defined above (see [9],[10]).

We shall seek the solutions $\mathcal{F}(z) \in \mathbb{C}_K[[z]]$ of the problem (8)–(9) in the form (17).

Theorem 3.1 *The formal Laurent series (17) is the solution of the differential problem (8)–(9) if and only if its Borel transformation (18) is the generating function of the solution $f(x)$ of the difference problem (15)–(16).*

Proof.

Necessity.

Using the linearity of the differential operator $P(D)$ and the definition of (4) for differentiating D^ω, we obtain:

$$P(D)\mathcal{F}(z) = \sum_{|\omega|_\nu \leqslant d} c_\omega D^\omega \sum_{x \in K \cap \mathbb{Z}^n} \frac{f(x) z^x}{\langle x, \alpha \rangle !} = \sum_{|\omega|_\nu \leqslant d} c_\omega \sum_{x - \omega \in K \cap \mathbb{Z}^n} \frac{\langle x, \alpha \rangle !}{\langle x - \omega, \alpha \rangle !} \frac{f(x) z^{x - \omega}}{\langle x, \alpha \rangle !}.$$

After standard transformations, the expression for $P(D)\mathcal{F}$ takes the form

$$P(D)\mathcal{F}(z) = \sum_{|\omega|_\nu \leqslant d} c_\omega \sum_{x \in K \cap \mathbb{Z}^n} \frac{f(x + \omega) z^x}{\langle x, \alpha \rangle !}.$$

Further, changing the order of summation in the last equality and taking into account the equation (8), we obtain:

$$P(D)\mathcal{F}(z) = \sum_{x\in K\cap\mathbb{Z}^n}\sum_{|\omega|_\nu\leqslant d}\frac{c_\omega f(x+\omega)z^x}{\langle x,\alpha\rangle!} = \sum_{x\in K\cap\mathbb{Z}^n}\frac{g(x)z^x}{\langle x,\alpha\rangle!}.$$

Finally, equating the coefficients of the same powers, we have

$$\sum_{|\omega|_\nu\leqslant d} c_\omega f(x+\omega) = g(x),$$

that is, $f(x)$ satisfies the difference equation (15).

In general, it is similarly proved that the condition (9) implies the condition (16)

$$D^{a^j k}[\mathcal{F}-\Phi] = D^{a^j k}\left[\sum_{x\in K\cap\mathbb{Z}^n}\frac{f(x)z^x}{\langle x,\alpha\rangle!} - \sum_{x\in K\cap\mathbb{Z}^n}\frac{\varphi(x)z^x}{\langle x,\alpha\rangle!}\right] =$$

$$= \sum_{x-a^jk\in K\cap\mathbb{Z}^n}\frac{\langle x,\alpha\rangle!(f(x)-\varphi(x))z^{x-a^jk}}{\langle x-a^jk,\alpha\rangle!\langle x,\alpha\rangle!} = \sum_{x\in K\cap\mathbb{Z}^n}\frac{(f(x+a^jk)-\varphi(x+a^jk))z^x}{\langle x,\alpha\rangle!}.$$

Using the definition of the shift operator δ_j, we obtain

$$D^{a^j k}[\mathcal{F}-\Phi] = \sum_{x\in K\cap\mathbb{Z}^n}\frac{\delta^{a^j k}(f(x)-\varphi(x))z^x}{\langle x,\alpha\rangle!},$$

further, using the condition (9), we obtain

$$\sum_{x\in K\cap\mathbb{Z}^n}\frac{\delta^{a^j k}(f(x)-\varphi(x))z^x}{\langle x,\alpha\rangle!}\Big|_{z^{a^j}=0} = 0.$$

According to the definition of (5), we have

$$\delta^{a^j k}[f(x)-\varphi(x)]|_{x\in\Gamma_j\cap\mathbb{Z}^n} = 0.$$

Sufficiency.

If we make the calculations from the proof of necessity in the reverse order, then we get (8) and (9) respectively from (15) and (16).

Proof of Theorem 1.

Since the condition (10) is satisfied, then by the above Theorem 1 from [5] on the solvability of the difference problem (15)–(16) there exists a unique solution $f(x)$. By Theorem 2, the Laurent series $\mathcal{F} = \sum\limits_{x\in K\cap\mathbb{Z}^n}\frac{f(x)z^x}{\langle x,\alpha\rangle!}$ is the (unique) solution of the problem (8)–(9).

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О формальных решениях начально-краевой задачи Хермандера в классе рядов Лорана

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Определяются дифференцирования кольца рядов Лорана с носителями в рациональных конусах и для полиномиальных дифференциальных операторов с постоянными коэффициентами доказывается теорема существования и единственности решения аналога одной начально-краевой задачи Хермандера в классе формальных рядов Лорана.

Ключевые слова: дифференциальный оператор, задача Хермандера, разностные уравнения, кратные ряды Лорана.